

4. THE THEORY OF THE DIPOLE ANTENNA

The theory of the dipole attracts attention of experts, basically, for two reasons. The first reason lies in the constructive similarity of the dipole antenna and the Hertz dipole. The radiation field of the dipole at limiting transition degenerates in a radiation field of the Hertz dipole, that further enhances the connection between the dipole antenna and the Hertz dipole.

The second reason is that the dipole finds extremely wide application in the range from decametric to centimetric waves. Its wide practical use is caused by both simplicity of the aerial and simplicity of its engineering calculation.

Consequently, the dipole is very well investigated experimentally and theoretically. It is possible to state its theory with sufficient completeness for engineering practice in a compact form without difficult mathematical operations. Its importance for the aerial engineering is cardinal: the dipole is used as an independent aerial as well as an element of antenna systems.

4.1. Radiation field of the dipole

The dipole comprises two metal rods of the identical form, placed on one axis, between which the generator of high frequency (Fig. 4.1) is included. The rods are characterized by radius a , length l and are known as the radiator arms. Usually, the cross size $2a$ is much less than the radiator length $2l$. At strict statement of a problem about the dipole radiation, it is necessary to start with the form of the radiator arms, Maxwells equations and limiting conditions. The decision of such problem meets the great difficulties, therefore we shall consider the approximate method of calculation.

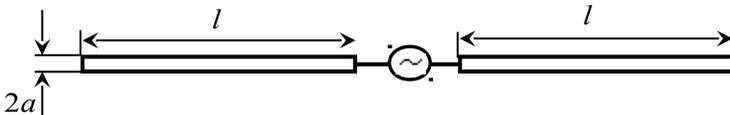


Fig. 4.1

This method supposes that the current distribution along the radiator is known. Therefore, it is possible to present radiator as a set of electric dipoles and to find a radiation field of the antenna as the sum of fields of elementary sources.

Let us take a two-wire open-ended line (Fig. 4.2(a)) and pull apart its ends (Fig. 4.2(b)) so that axes of both conductors have coincided (Fig. 4.2(c)). It is obvious, that we shall proceed from the two-wire line to the dipole. Let us assume, that current distribution will not change. Then the amplitude of a current I_z in the dipole section taken on a distance Z from the point of the generator connection, like in the two-wire open-ended line, is determined by the expression

$$I_z = \begin{cases} \dot{I}_L \sin k(l+z) & \text{at } z \leq 0, \\ \dot{I}_L \sin k(l-z) & \text{at } z \geq 0, \end{cases} \quad (4.1)$$

where \dot{I}_L is the complex amplitude of current in the loop (antinode).

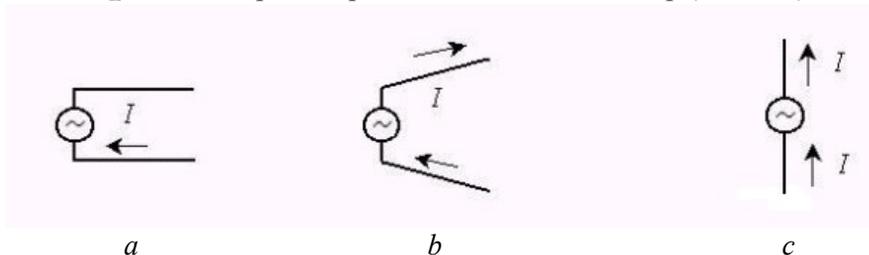


Fig. 4.2

This assumption, as experimental researches and the strict theory of the dipole show, will be the more accurate, the smaller dipole radius. With increase of radius the current distribution along the dipole verges towards the voltage distribution in a line with a running wave of current.

The basic difference of the dipole from a two-wire line is that the dipole is the radiating system, and the two-wire line practically does not radiate electromagnetic waves and its field is potential. In this connection the concept of voltage with reference to a dipole should be used rather cautiously. For the dipole, which sizes are commensurable with the wavelength, the charge distribution is considered instead of the voltage distribution.

Proceeding from the equation of continuity

$$\operatorname{div} j^e + \frac{\partial \rho^e}{\partial t} = 0,$$

and also assuming that both current and charge are uniformly distributed over the dipole cross-section

$$j^e = \frac{I_z}{S} \quad \text{and} \quad \dot{\rho}^e = \frac{\dot{\tau}_z}{S},$$

we obtain

$$\frac{\partial \dot{I}_z}{\partial z} = -i\omega \dot{\tau}_z, \quad (4.2)$$

where $\dot{\tau}_z$ is the linear density of charge on distance Z from the point of generator connection.

From the equations (4.1) and (4.2) the linear charge density in any section of the radiator can be found:

$$\dot{\tau}_z = \begin{cases} \frac{ik\dot{I}_L \cos k(l+z)}{\omega} & \text{for } z \leq 0; \\ -\frac{ik\dot{I}_L \cos k(l-z)}{\omega} & \text{for } z \geq 0. \end{cases}$$

(4.3)

The distribution of charge along the dipole, as follows from the formula (4.3), corresponds to the voltage distribution in a two-wire loss-free line.

To investigate a radiation field let us separate out elementary sources (points A and B in Fig. 4.3) in the dipole located at arbitrary distance Z from terminals. The length of elementary radiators dz is chosen so that the amplitude of

practically identical. The design number 1, and an element in 1 elements to the observation r_1 and r_2 , respectively, centre to the observation point M lies in a far- z parallel to each other.

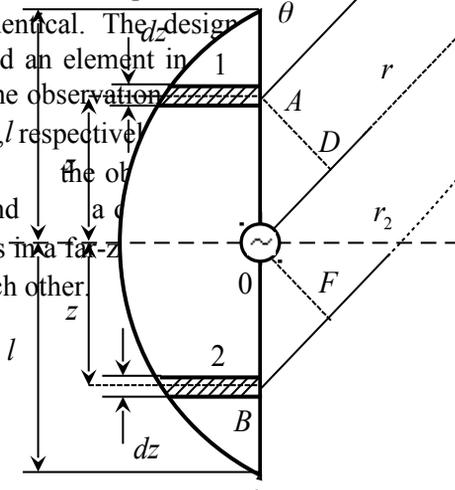


Fig.4.3

The field intensity of the first elementary source in point M from formula (3.1) is

$$d\dot{E}_1 = i \frac{\dot{I}_z W dz}{2\lambda r_1} \sin \theta e^{-i kr_1} .$$

The field intensity of the second elementary source is

$$d\dot{E}_2 = i \frac{\dot{I}_z W dz}{2\lambda r_2} \sin \theta e^{-i kr_2} .$$

Summarizing these intensities we get:

$$d\dot{E} = d\dot{E}_1 + d\dot{E}_2 = i \frac{\dot{I}_z W dz}{2\lambda} \sin \theta \left(\frac{e^{-i kr_1}}{r_1} + \frac{e^{-i kr_2}}{r_2} \right) . \quad (4.4)$$

According to expression (1.14) it is assumed that $r = r_1 = r_2$. Therefore, amplitudes of vectors dE_1 and dE_2 may be considered identical. An increment of distances can be found for phase multipliers from triangles AOD and BOF and, as follows from the equation (1.15), we may write down, that

$$r_1 = r - z \cos \theta;$$

$$r_2 = r + z \cos \theta.$$

Then expression (4.4) becomes simpler:

$$d\dot{E} = i \frac{\dot{I}_z W dz}{2\lambda r} \sin \theta e^{-ikr} (e^{ikz \cos \theta} + e^{-ikz \cos \theta}). \quad (4.5)$$

Taking into account

$$0.5(e^{i\varphi} + e^{-i\varphi}) = \cos \varphi,$$

and current value I_z [equation (4.1)], expression (4.5) can be rewritten

$$d\dot{E} = i \frac{\dot{I}_L W \sin \theta}{\lambda r} e^{-ikr} \sin k(l - z) \cos(kz \cos \theta) dz.$$

The field intensity, induced by the whole dipole antenna, is found after integration over the arm length

$$\dot{E} = i \frac{\dot{I}_L W \sin \theta}{\lambda r} e^{-ikr} \int_0^l \sin k(l - z) \cos(kz \cos \theta) dz. \quad (4.6)$$

Carrying out integration, we, finally, obtain

$$\dot{E} = i \frac{\dot{I}_L W}{2\pi r} \frac{\cos(kl \cos \theta) - \cos kl}{\sin \theta} e^{-ikr}. \quad (4.7)$$

Like formula (1.19), expression (4.7) consists of the amplitude factor, the DC and the phase multiplier.

From equation (4.1), current at $z = 0$ is

$$\dot{I}_A = \dot{I}_L \sin kl. \quad (4.8)$$

Using current value (4.8), let us determine from expression (4.7) the amplitude multiplier at the unnormalized DC, taking into consideration the value of the wave impedance in a free space

$$E_{max H} = \frac{60 I_A}{r \sin kl}.$$

Unnormalized DC of the dipole is

$$f(\theta) = \frac{\cos(kl \cos \theta) - \cos kl}{\sin \theta}. \quad (4.9)$$

It is seen from expression (4.9), that the distribution of the dipole's field in space does not depend on angle φ , therefore the spatial DD

represents a surface formed by rotation a curve around a polar axis. In the azimuthal plane the DD takes form of a circle.

The field distribution in space is determined by the relative length of the dipole l/λ . If the relative length is small $kl \ll 1$, expression (4.7), in view of (4.8), transforms to formula (3.1). Therefore, for short radiators, DD appears as in Fig. 3.2. For the half-wave dipole $l = 0.25\lambda$, which is widely used

$$F(\theta) = \frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta}.$$

The DD of the half-wave dipole is represented in Fig. 4.4(a). The DD's lobes are slightly narrow in comparison with lobes of the Hertz dipole DD (Fig.3.2).

The beam width of the half-wave dipole is

$$2\theta_0 = 180^\circ \text{ and } 2\theta_{0.5} = 80^\circ.$$

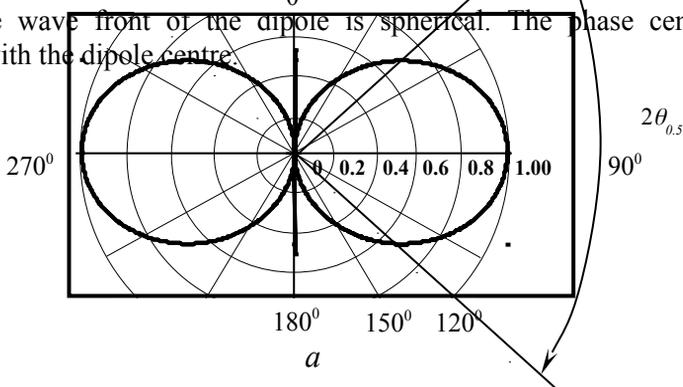
The beam width of the wave dipole is

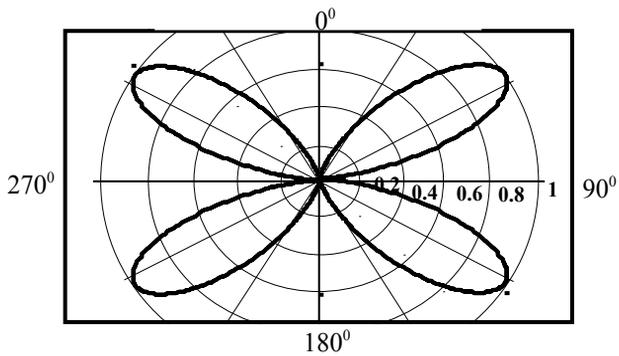
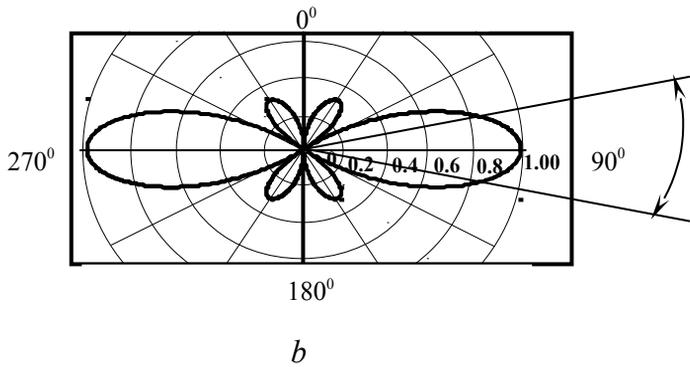
$$2\theta_0 = 180^\circ \text{ and } 2\theta_{0.5} = 44^\circ.$$

On further lengthening of the radiator, the main lobe is still getting narrow and the side lobes appear. So, for a dipole with the arm $l = 0.625\lambda$ the direction of the maximum radiation is still perpendicular to the dipole axis, but the side lobes have already appeared (Fig. 4.4(b)). At this the beam width: $2\theta_0 = 74^\circ$ and $2\theta_{0.5} = 31^\circ$.

The further lengthening of the dipole results in the reduction of the radiation intensity in the direction perpendicular to the aerial axis. This intensity will be smaller than the intensity in the direction of maximum of side lobes. Finally, at length $l = \lambda$ the radiation in the azimuthal plane i
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 the maximal radiation i

The wave front of the dipole is spherical. The phase centre coincides with the dipole centre.





4.2. Basic parameters of the dipole

Fig. 4.4.

One of the main parameters of the dipole is the radiation resistance. For its determination it is necessary to find radiation power P_{Σ} . Let us substitute in formula (1.27) the value of the amplitude of the field intensity from equation (4.7) at $W_c = 120\pi$ Ohm

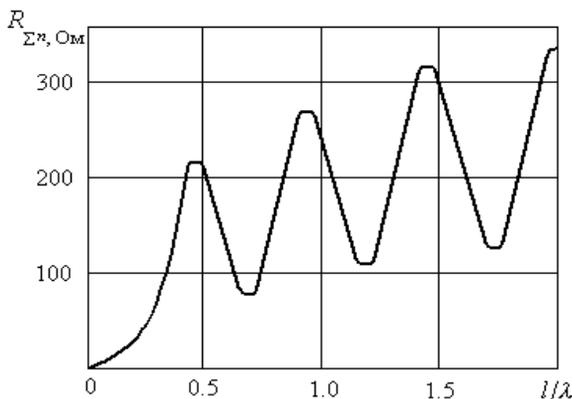
$$P_{\Sigma} = \frac{15I_L^2 \pi^2}{\pi} \int_0^{\pi} \int_0^{2\pi} \frac{[\cos(kl \cos\theta) - \cos kl]^2 d\theta d\varphi}{\sin\theta}. \quad (4.10)$$

On integration, it is possible to calculate the radiation resistance by formula (2.16):

$$R_{\Sigma L} = 30[(Si\ 4kl - 2Si\ 2kl)\sin\ 2kl + (C + \ln\ kl + Ci\ 4kl - 2Ci\ 2kl)\cos\ 2kl + 2(C + \ln\ 2kl - Ci\ 2kl)],$$

where $Si\ x$ is integral sine from argument x ; $Ci\ x$ is integral cosine from argument x ; $C = 0.577$ is the Euler constant.

The dependence of the radiation resistance $R_{\Sigma n}$ on the relative dipole length is shown in Fig. 4.5. The character of the resistance change is determined by the phase distribution of current along the dipole. At small lengths of dipole arms the current phase in all points is the same and, consequently, with growing of length l the radiation resistance increases. At $2l > \lambda$, there are regions with the antiphase currents that



reduces both radiation power and radiation resistance $R_{\Sigma n}$. As the current phase reverses every 0.5λ length of the arm, the change of the

radiation resistance has an oscillatory character. For the half-wave dipole $R_{\Sigma L} = 73.1 \text{ Ohm}$, for the wave radiator - $R_{\Sigma L} = 200 \text{ Ohm}$.

Applying the method of induced EMF we can also determine a reactive component of the radiation impedance. Thus, for the half-wave dipole

$$Z_{\Sigma L} = (73.1 + i42.5) \text{ Ohm}.$$

For compensation of the reactive component it is necessary to shorten the half-wave dipole length by 5-7 %.

With the help of the lossy long lines theory the approximate expression for the dipole input impedance can be obtained:

$$Z_A = \frac{R_{\Sigma L}}{\frac{R_{\Sigma L}^2}{W_A^2} + \sin^2 kl} - i \frac{W_A}{2} \frac{\sin 2kl}{\frac{R_{\Sigma L}^2}{W_A^2} + \sin^2 kl},$$

where $W_A = 120 \left(\ln \frac{l}{a} - 1 \right)$ is the wave resistance of the dipole; a is the radius of the dipole.

For short dipoles ($l < 0.4\lambda$) the following inequality is valid

$$R_{\Sigma L} / W_A < \sin kl.$$

Therefore, the input impedance of the short radiator can be defined by the simplified formula:

$$Z_A = \frac{R_{\Sigma L}}{\sin^2 kl} - i W_A \operatorname{ctg} kl. \quad (4.11)$$

The dependence of the active component of the input impedance from the dipole length is represented in Fig. 4.6(a), the dependence of the reactive component - in Fig. 4.6(b).

Proceeding from the character of the input impedance change (Fig. 4.6) we can make a conclusion, that the dipole behaves as an oscillatory contour. At $l = 0.25\lambda$, as follows from formula (4.11),

$$R_A = R_{\Sigma L} = 73.1 \text{ Ohm}.$$

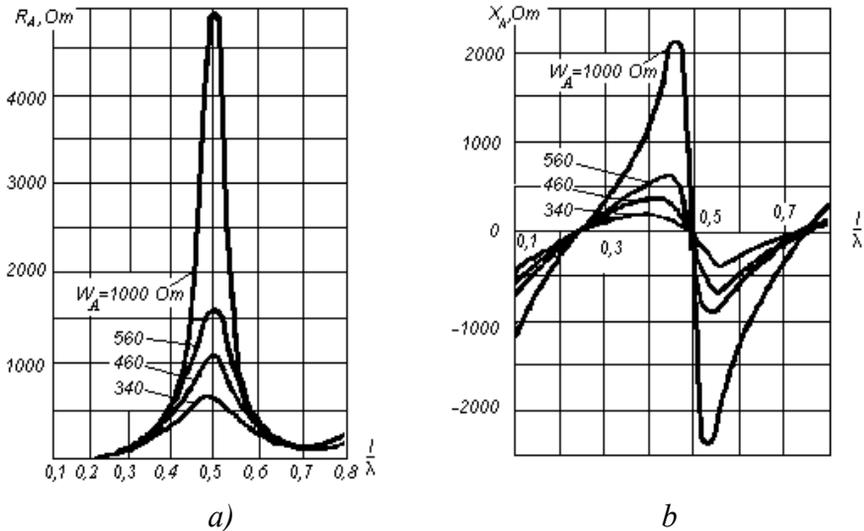


Fig. 4.6

The maximal wave-length, at which the dipole input impedance is only active, is referred to as the own wave-length of the dipole λ_w . It is obvious, that

$$\lambda_w = 4l.$$

At $l = 0.5\lambda$, the reactive component of the input impedance also moves to zero. Thus

$$R_A = \frac{W_A^2}{R_{\Sigma L}}.$$

As seen in Fig. 4.6, the input impedance essentially depends on the wave resistance of the dipole. The lower the value of the wave resistance, the narrower the borders of active and reactive components of the impedance change at wave-length variations. This circumstance is used when designing wide-band aerials.

Let us find the directivity factor (DF) for the dipole with the direction of the maximal radiation, which is perpendicular to the dipole's axis. Taking in formula (4.7) $\theta = 90^\circ$ and $W = 120\pi \text{ Ohm}$, we obtain

$$E_{max} = \frac{60I_L}{r}(1 - \cos kl). \quad (4.12)$$

Substituting values of the amplitude (4.12) in formula (2.29) and taking into account expression (2.15), we obtain DF of the dipole:

$$D = \frac{120}{R_{\Sigma L}}(1 - \cos kl)^2.$$

In the case of the half-wave radiator $D = 1.64$, for the wave dipole $D = 2.4$. The maximal value of DF of the dipole is reached at $l = 0.625\lambda$. In this case $R_{\Sigma L} = 110 \text{ Ohm}$ and $D = 3.1$. Substituting in formula (2.41) value of the maximal field intensity (4.12) and taking into account expression (4.8), we find the dipole effective length:

$$l_e = \frac{2}{k} \frac{1 - \cos kl}{\sin kl} = \frac{\lambda}{\pi} \operatorname{tg} \frac{kl}{2}. \quad (4.13)$$

For dipoles of small length $l \ll \lambda$ the tangent argument will be also small, therefore, we can replace the tangent by its argument. Thus, the effective length is determined from

$$l_e \cong \frac{kl\lambda}{2\pi} \approx l.$$

The effective length of the half-wave dipole, as it follows from equation (4.13), is equal to λ/π .